Quite suddenly and without warning, Herb fell victim to the old adage, “If you don’t use it, you lose it.”
Mathematics Placement

• The ACT COMPASS math test is a self-adaptive test, which potentially tests students within four different levels of math including pre-algebra, algebra, college algebra, and trigonometry. As you answer questions correctly, you will move into more difficult levels of math. Similarly, if you answer questions incorrectly, the computerized test will begin to ask questions from a lower level of math.

• Multiple-choice items in each of the five mathematics placement areas test the following:
  – **basic skills**—performing a sequence of basic operations
  – **application**—applying sequences of basic operations to novel settings or in complex ways
  – **analysis**—demonstrating conceptual understanding of principles and relationships in mathematical operations

• Students are permitted to use approved calculators when completing the COMPASS® mathematics placement or diagnostic tests. An online calculator is available for those students who wish to access it via Microsoft Windows.

• Because this is an adaptive test, you may change your answer while you are still on a problem, but once you go on to another problem, you may not go back to a question.
Mathematics Placement
Sample Questions (Geometry)

• Following are 11 sample College Geometry Placement Test Questions taken from the ACT COMPASS website.

• First you will see the question, then the following slide will have the answer.

• If you need some additional refreshers, the remainder of the slides cover the content from the Geometry section.
Geometry Placement Test

• Primary content areas included in the Geometry Placement Test include:
  • Triangles (perimeter, area, Pythagorean theorem, etc.)
  • Circles (perimeter, area, arcs, etc.)
  • Angles (supplementary, complementary, adjacent, vertical, etc.)
  • Rectangles (perimeter, area, etc.)
  • Three-dimensional concepts
  • Hybrid (composite) shapes
1. In the figure below, line \( m \) is parallel to line \( n \), and line \( t \) is a transversal crossing both \( m \) and \( n \). Which of the following lists has 3 angles that are all equal in measure?

A. \( \angle a, \angle b, \angle d \)
B. \( \angle a, \angle c, \angle d \)
C. \( \angle a, \angle c, \angle e \)
D. \( \angle b, \angle c, \angle d \)
E. \( \angle b, \angle c, \angle e \)
Geometry Placement Test
Sample Questions

1. In the figure below, line \( m \) is parallel to line \( n \), and line \( t \) is a transversal crossing both \( m \) and \( n \). Which of the following lists has 3 angles that are all equal in measure? 

\[ \angle a, \angle b, \angle d \]
\[ \angle a, \angle c, \angle d \]
\[ \angle a, \angle c, \angle e \]
\[ \angle b, \angle c, \angle d \]
\[ \angle b, \angle c, \angle e \]

This is an example of Special Angles. The correct answer is A. See [Special Angles slides](#) for additional information on this topic.

To solve: Vertical angles are congruent. Alternate interior angles are congruent.
2. As shown in the figure below, \( \triangle ABC \) is isosceles with the length of \( \overline{AB} \) equal to the length of \( \overline{AC} \). The measure of \( \angle A \) is 40° and points \( B, C, \) and \( D \) are collinear. What is the measure of \( \angle ACD \)?

\[ \begin{array}{c}
\text{A. } 70^\circ \\
\text{B. } 80^\circ \\
\text{C. } 110^\circ \\
\text{D. } 140^\circ \\
\text{E. } 160^\circ 
\end{array} \]
Geometry Placement Test
Sample Questions

2. As shown in the figure below, $\triangle ABC$ is isosceles with the length of $\overline{AB}$ equal to the length of $\overline{AC}$. The measure of $\angle A$ is $40^\circ$ and points $B, C, \text{ and } D$ are collinear. What is the measure of $\angle ACD$?

[Diagram of an isosceles triangle with angles labeled and a question mark for the unknown angle $\angle ACD$]

This is an example of Isosceles Triangles. The correct answer is C. See [Triangles slides] for additional information on this topic.

To solve: 

$m\angle ABC = m\angle BCA = (180 - 40) \div 2 = 70$  \hspace{1cm} \text{Angles opposite congruent sides are congruent}

$m\angle LACD = 180 - 70 = 110$
3. The diagram below shows a pasture which is fenced in. All but 1 section of fence run straight north-south or east-west. Consecutive fence posts are 10 feet apart except for the 1 diagonal section. Which of the following statements best describes $P$, the perimeter of the pasture, in feet?

A. $P > 210$
B. $P = 210$
C. $P < 210$
D. $P > 230$
E. $P = 240$
3. The diagram below shows a pasture which is fenced in. All but 1 section of fence run straight north-south or east-west. Consecutive fence posts are 10 feet apart except for the 1 diagonal section. Which of the following statements best describes $P$, the perimeter of the pasture, in feet?

**A.** $P > 210$

**B.** $P = 210$

**C.** $P < 210$

**D.** $P > 230$

**E.** $P = 240$

This is an example of Perimeter of Hybrid Shapes. The correct answer is A. See Perimeter slides for additional information on this topic.

To solve: There are 20 regular 10 feet sections plus one diagonal section which is greater than 10 feet. Therefore, the total perimeter (distance around) is $P > 210$. 
4. A person had a rectangular-shaped garden with sides of lengths 16 feet and 9 feet. The garden was changed into a square design with the same area as the original rectangular-shaped garden. How many feet in length are each of the sides of the new square-shaped garden?

A. 7
B. 9
C. 12
D. $5\sqrt{7}$
E. 16
Geometry Placement Test
Sample Questions

4. A person had a rectangular-shaped garden with sides of lengths 16 feet and 9 feet. The garden was changed into a square design with the same area as the original rectangular-shaped garden. How many feet in length are each of the sides of the new square-shaped garden?

A. 7
B. 9
C. 12
D. $5\sqrt{7}$
E. 16

This is an example of Area of Rectangles. The correct answer is C. See Area slides for additional information on this topic.

To solve: Area (rectangle) = length x width = 16 x 9 = 144 sq. feet
Area (square) = side²
144 = s²
s = 12 feet
5. In the figure below, \( \Delta ABC \) is a right triangle. The length of \( \overline{AB} \) is 6 units and the length of \( \overline{CB} \) is 3 units. What is the length, in units, of \( \overline{AC} \) ?

\[ \begin{array}{c}
\text{A.} \ 5 \\
\text{B.} \ 3\sqrt{3} \\
\text{C.} \ 3 + \sqrt{5} \\
\text{D.} \ 3\sqrt{5} \\
\text{E.} \ 3\sqrt{6} \\
\end{array} \]
Geometry Placement Test
Sample Questions

5. In the figure below, \( \triangle ABC \) is a right triangle. The length of \( \overline{AB} \) is 6 units and the length of \( \overline{CB} \) is 3 units. What is the length, in units, of \( \overline{AC} \)?

![Diagram of right triangle with sides labeled A, B, and C, with AB = 6 units, BC = 3 units, and AC as the hypotenuse.]

A. 5
B. \( 3\sqrt{3} \)
C. \( 3 + \sqrt{5} \)
D. \( 3\sqrt{5} \)
E. \( 3\sqrt{6} \)

This is an example of Triangles. The correct answer is B. See Triangles slides for additional information on this topic.

To solve: Use the Pythagorean Theorem: \( a^2 + b^2 = c^2 \)

\[
3^2 + b^2 = 6^2 \\
9 + b^2 = 36 \\
b^2 = 27 \\
b = \sqrt{27} = \sqrt{(9 \times 3)} = 3\sqrt{3}
\]
6. If a central angle of measure $30^\circ$ is subtended by a circular arc of length 6 meters, as is illustrated below, how many meters in length is the radius of the circle?

A. $\frac{\pi}{36}$
B. $\frac{1}{5}$
C. $\pi$
D. $\frac{36}{\pi}$
E. 180
Geometry Placement Test
Sample Questions

6. If a central angle of measure $30^\circ$ is subtended by a circular arc of length 6 meters, as is illustrated below, how many meters in length is the radius of the circle?

This is an example of Arc of a Circle. The correct answer is D. See Circle slides for additional information on this topic.

To solve: The length of an arc $= \left(\frac{m}{360}\right) \times 2\pi r$, where $m$ is the degree measure of the central angle. Substitute value of central angle and length of arc then solve for $r$:

\[
6 = \left(\frac{30}{360}\right) \times 2\pi r \\
6 = \left(\frac{1}{12}\right) \times 2\pi r \\
6 = \left(\frac{1}{6}\right)\pi r \\
36/\pi = r
\]
7. A rectangular box with a base 2 inches by 6 inches is 10 inches tall and holds 12 ounces of breakfast cereal. The manufacturer wants to use a new box with a base 3 inches by 5 inches. How many inches tall should the new box be in order to hold exactly the same volume as the original box? (Note: The volume of a rectangular box may be calculated by multiplying the area of the base by the height of the box.)

A. 8  
B. 9  
C. 10  
D. 11  
E. 12
7. A rectangular box with a base 2 inches by 6 inches is 10 inches tall and holds 12 ounces of breakfast cereal. The manufacturer wants to use a new box with a base 3 inches by 5 inches. How many inches tall should the new box be in order to hold exactly the same volume as the original box? (Note: The volume of a rectangular box may be calculated by multiplying the area of the base by the height of the box.)

A. 8 
B. 9 
C. 10 
D. 11 
E. 12

This is an example of Three-Dimensional Concepts. The correct answer is A. See 3-D Shapes slides for additional information on this topic.

To solve: Volume = length x width x height

V(box 1) = 2 x 6 x 10 = 120 cubic inches
V(box 2) = 3 x 5 x h = 120

\[15h = 120\]
\[h = 8\]
8. In the figure below, the circle centered at $B$ is internally tangent to the circle centered at $A$. The smaller circle passes through the center of the larger circle and the length of $\overline{AB}$ is 5 units. If the smaller circle is cut out of the larger circle, how much of the area, in square units, of the larger circle will remain?

A. $10\pi$
B. $25\pi$
C. $75\pi$
D. $100\pi$
E. $300\pi$
Geometry Placement Test
Sample Questions

8. In the figure below, the circle centered at B is internally tangent to the circle centered at A. The smaller circle passes through the center of the larger circle and the length of $\overline{AB}$ is 5 units. If the smaller circle is cut out of the larger circle, how much of the area, in square units, of the larger circle will remain?

This is an example of Area of Circle. The correct answer is C. See Circle slides for additional information on this topic.

To solve: Area (circle $A$) = $\pi r^2 = \pi 10^2 = 100 \pi$
Area (square $B$) = $\pi r^2 = \pi 5^2 = 25 \pi$
Difference is $100 \pi - 25 \pi = 75 \pi$
9. In the figure below, $\overline{AB}$ and $\overline{CD}$ are parallel, and lengths are given in units. What is the area, in square units, of trapezoid $ABCD$?

A. 36  
B. 52  
C. 64  
D. 65  
E. 104
9. In the figure below, $AB$ and $CD$ are parallel, and lengths are given in units. What is the area, in square units, of trapezoid $ABCD$?

To solve: Area (trapezoid) = $\frac{1}{2}h(b_1 + b_2)$. First we need to determine the length of segment $DC$. Using the work above, $DC = 16$. Now substitute values into the formula: Area (trapezoid) = $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(4)(10 + 16) = \frac{1}{2}(4)(26) = 52$
10. A 6-foot spruce tree is planted 15 feet from a lighted streetlight whose lamp is 18 feet above the ground. How many feet long is the shadow of that tree?

A. 5.0  
B. 7.5  
C. 7.8  
D. 9.6  
E. 10.0
This is an example of Similar Triangles. The correct answer is B. See Similar Triangles slides for additional information on this topic.

To solve: Corresponding sides of similar triangles are proportional to each other.
11. In the figure below, the lengths of $\overline{DE}$, $\overline{EF}$, and $\overline{FG}$ are given, in units. What is the area, in square units, of $\triangle DEG$?

![Diagram of a triangle with coordinates D(0,0), E(7,0), F(12,0), and G(0,10).]

A. 29
B. 47.5
C. 60
D. $6\sqrt{149}$
E. 120
11. In the figure below, the lengths of $\overline{DE}$, $\overline{EF}$, and $\overline{FG}$ are given, in units. What is the area, in square units, of $\Delta DEG$?

![Diagram of triangle](image)

A. 29
B. 47.5
C. 60
D. $6\sqrt{149}$
E. 120

This is an example of Area of Triangles. The correct answer is C. See [Area slides](#) for additional information on this topic.

To solve: $\text{Area(triangle)} = \frac{1}{2}bh = \frac{1}{2}(12)10 = 60$ square units
Geometry Review

- The following slides review the concepts found on the COMPASS Geometry Placement Test.
Geometry Placement Test

Primary content areas included in the Geometry Placement Test include:

- Triangles (perimeter, area, Pythagorean theorem, etc.)
- Circles (perimeter, area, arcs, etc.)
- Angles (supplementary, complementary, adjacent, vertical, etc.)
- Rectangles (perimeter, area, etc.)
- Three-dimensional concepts
- Hybrid (composite) shapes
<table>
<thead>
<tr>
<th>Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <strong>Circle</strong>: Refer to questions 4, 7, 8, 11, and 14-17</td>
</tr>
<tr>
<td>o Radius = ( r = \frac{d}{2} )</td>
</tr>
<tr>
<td>o Diameter = ( d = 2r )</td>
</tr>
<tr>
<td>o Area = ( \pi r^2 )</td>
</tr>
<tr>
<td>o Circumference = ( 2\pi r = \pi d )</td>
</tr>
<tr>
<td>o All radii of the same circle are equal.</td>
</tr>
<tr>
<td><img src="circle.png" alt="Circle Diagram" /></td>
</tr>
<tr>
<td>• <strong>Square</strong>: Refer to questions 3 and 15</td>
</tr>
<tr>
<td>o Side = ( s )</td>
</tr>
<tr>
<td>o Area = ( s \times s )</td>
</tr>
<tr>
<td><img src="square.png" alt="Square Diagram" /></td>
</tr>
<tr>
<td>• <strong>Rectangle</strong>: Refer to questions 1, 2, 19, and 20</td>
</tr>
<tr>
<td>o Length = ( l )</td>
</tr>
<tr>
<td>o Width = ( w )</td>
</tr>
<tr>
<td>o Area = ( l \times w )</td>
</tr>
<tr>
<td>o Both diagonals of a rectangle are equal.</td>
</tr>
<tr>
<td><img src="rectangle.png" alt="Rectangle Diagram" /></td>
</tr>
<tr>
<td>• <strong>Triangle</strong>: Refer to questions 1-6, and 8-11</td>
</tr>
<tr>
<td>o Base = ( b )</td>
</tr>
<tr>
<td>o Height = ( h )</td>
</tr>
<tr>
<td>o Area = ( \frac{1}{2} bh )</td>
</tr>
<tr>
<td><img src="triangle.png" alt="Triangle Diagram" /></td>
</tr>
<tr>
<td>• <strong>Trapezoid</strong>: Refer to question 18</td>
</tr>
<tr>
<td>o Larger Base = ( B )</td>
</tr>
<tr>
<td>o Smaller Base = ( b )</td>
</tr>
<tr>
<td>o Height = ( h )</td>
</tr>
<tr>
<td>o Area = ( \frac{1}{2} h(B + b) )</td>
</tr>
<tr>
<td><img src="trapezoid.png" alt="Trapezoid Diagram" /></td>
</tr>
</tbody>
</table>

*The perimeter of any figure is the sum of its sides.*
Triangles

1. Classification
2. Perimeter
3. Area
4. Pythagorean theorem
5. Congruency
6. Similar Triangles
Geometry

<table>
<thead>
<tr>
<th>Definition</th>
<th>Representation</th>
</tr>
</thead>
</table>
| **A point** is an exact location in space that has position but no dimensions. It is named with a capital letter. | • P  
Name: point P |
| **A line** is an infinite set of points that extends in two directions with no endpoints. It has an infinite dimension of length. It is named by any two points on the line or a single lowercase letter. |  
Name: line S, line AB, or $AB$ |
| **A plane** is a set of three or more points forming a flat surface with no boundaries. It has two infinite dimensions of width and length. It is named by any three points in the plane or a capital script letter. (In a drawing, a plane is shown with boundaries so it can be seen.) |  
Name: plane CDE or plane R |
| **A ray** is an endpoint and all the points extending indefinitely in one direction from that endpoint. It is named by the endpoint and a point on the ray. |  
Name: ray FG or $\overrightarrow{FG}$ |
| **A line segment** is part of a line defined by two endpoints. It is named by any two points on the line segment. |  
Name: segment HI or $\overline{HI}$ |
# Angles

<table>
<thead>
<tr>
<th>Angle</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>An <strong>acute angle</strong> is an angle less than 90.</td>
<td><img src="angle55.png" alt="55°" /></td>
</tr>
<tr>
<td>A <strong>right angle</strong> is an angle that measures exactly 90 and is denoted by a box at the vertex.</td>
<td><img src="angle90.png" alt="90°" /></td>
</tr>
<tr>
<td>An <strong>obtuse angle</strong> is an angle greater than 90.</td>
<td><img src="angle120.png" alt="120°" /></td>
</tr>
<tr>
<td>A <strong>straight angle</strong> is an angle that measures 180.</td>
<td><img src="angle180.png" alt="180°" /></td>
</tr>
</tbody>
</table>
## Angles

<table>
<thead>
<tr>
<th>Angle</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Complementary angles</strong></td>
<td>are 2 angles whose sum is 90.</td>
</tr>
<tr>
<td>$\angle 1$ and $\angle 2$ are complementary.</td>
<td><img src="image1.png" alt="Complementary Angles" /></td>
</tr>
<tr>
<td><strong>Supplementary angles</strong></td>
<td>are 2 angles whose sum is 180. (A straight angle can be made up of supplementary angles.)</td>
</tr>
<tr>
<td>$\angle 3$ and $\angle 4$ are supplementary.</td>
<td><img src="image2.png" alt="Supplementary Angles" /></td>
</tr>
<tr>
<td><strong>Adjacent angles</strong></td>
<td>are angles that have the same vertex and share a side.</td>
</tr>
<tr>
<td>$\angle 5$ and $\angle 6$ are adjacent.</td>
<td><img src="image3.png" alt="Adjacent Angles" /></td>
</tr>
<tr>
<td><strong>Congruent angles</strong></td>
<td>are angles that have equal measures. The symbol for congruence is $\cong$.</td>
</tr>
<tr>
<td>$\angle A$ and $\angle B$ are congruent.</td>
<td>$\angle A \cong \angle B$</td>
</tr>
<tr>
<td><strong>Vertical angles</strong></td>
<td>are <em>congruent</em> angles that are formed by intersecting lines. They appear opposite of each other.</td>
</tr>
<tr>
<td>$\angle 7$ and $\angle 8$ are vertical angles.</td>
<td><img src="image4.png" alt="Vertical Angles" /></td>
</tr>
</tbody>
</table>
## Triangles

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>An <strong>acute triangle</strong> has three angles and each measures less than 90.</td>
<td>![40°, 70°, 70° triangle]</td>
</tr>
<tr>
<td>A <strong>right triangle</strong> has one angle equal to 90 (marked with the box). The side opposite the right angle is always the longest side and is called the hypotenuse.</td>
<td>![Right triangle with hypotenuse]</td>
</tr>
<tr>
<td>An <strong>obtuse triangle</strong> has one angle that is greater than 90.</td>
<td>![100° triangle]</td>
</tr>
<tr>
<td>An <strong>equiangular triangle</strong> has three angles that are equal in measure. (Since the sum of the angles in a triangle is 180, all angles in this triangle measure 60.)</td>
<td>![Equiangular triangle]</td>
</tr>
<tr>
<td>A <strong>scalene triangle</strong> has no <em>sides</em> that are the equal in length.</td>
<td>![Scalene triangle]</td>
</tr>
<tr>
<td>An <strong>isosceles triangle</strong> has two <em>sides</em> that are equal in length.</td>
<td>![Isosceles triangle]</td>
</tr>
<tr>
<td>An <strong>equilateral triangle</strong> has all three <em>sides</em> equal in length. NOTE: All equiangular triangles are also equilateral and vice versa.</td>
<td>![Equilateral triangle]</td>
</tr>
</tbody>
</table>

The sum of all of the interior angles of a triangle is 180.

\[ m\angle A + m\angle B + m\angle C = 180^\circ \]
The triangle must be both **Isosceles** (at least 2 sides & 2 angles equal) and **Obtuse** (one angle greater than 90 degrees).

Which of the following triangles is included in the shaded region?

A. Acute, Equilateral

B. Correct
   - Obtuse, Isosceles

C. Acute, Isosceles

D. Obtuse, Scalene
## Triangles

<table>
<thead>
<tr>
<th>Terms</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>An <strong>altitude</strong> is a line segment that connects any vertex to the opposite side at a right angle. (In the area formula for a triangle, the altitude is called the height.)</td>
<td><img src="image1" alt="Altitude" /></td>
</tr>
<tr>
<td>There are three altitudes in a triangle. The altitudes are concurrent because they share one point in common where they intersect. The point of concurrency is called the <strong>orthocenter</strong> of the triangle.</td>
<td></td>
</tr>
<tr>
<td>A <strong>median</strong> is a line segment that connects any vertex to the <strong>midpoint</strong> of the opposite side.</td>
<td><img src="image2" alt="Median" /></td>
</tr>
<tr>
<td>There are three medians in a triangle. The medians are concurrent because they share one point in common where they intersect. The point of concurrency is called the <strong>centroid</strong> of the triangle.</td>
<td></td>
</tr>
<tr>
<td>An <strong>angle bisector</strong> is a line segment that connects any vertex to the opposite side so that it divides the vertex angle into two equal angles.</td>
<td><img src="image3" alt="Angle Bisector" /></td>
</tr>
<tr>
<td>There are three angle bisectors in a triangle. The angle bisectors are concurrent because they share one point in common where they intersect. The point of concurrency is called the <strong>incenter</strong> of the triangle.</td>
<td></td>
</tr>
<tr>
<td>A <strong>perpendicular bisector</strong> is a line or line segment that passes through the midpoint of the side of the triangle and is perpendicular to that side.</td>
<td><img src="image4" alt="Perpendicular Bisector" /></td>
</tr>
<tr>
<td>There are three perpendicular bisectors in a triangle. The perpendicular bisectors are concurrent because they share one point in common where they intersect. The point of concurrency is called the <strong>circumcenter</strong> of the triangle.</td>
<td></td>
</tr>
</tbody>
</table>
Perimeter

- **Perimeter** – the distance around the outside of a figure. To find perimeter, add together the lengths of all the sides of the figure.

**Example**

- Find the perimeter of the figure below, simply add the lengths of all sides.

- Perimeter = 5 + 12 + 13 = 30 feet
Distance Formula

To find the distance between two points on the coordinate plane, use the **distance formula**. A line segment can be drawn connecting those two points. The distance between the two points is the same as the length of that line segment.

For point_1_ \((x_1, y_1)\) and point_2_ \((x_2, y_2)\), \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

Example:

Use the distance formula to find the length of line segment AB.
The coordinates for point A are (2, 4) and for point B are (6, 1).

Either point can be point \(_1\) or point \(_2\) in the distance formula. Pick one and be consistent throughout. If you start with point A to subtract the \(x\)-coordinates, then start with point A to subtract the \(y\)-coordinates.

Let A be point \(_1\) and B be point \(_2\). Plug the values into the equation.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ AB = \sqrt{(6 - 2)^2 + (1 - 4)^2} \]

\[ AB = \sqrt{(4)^2 + (-3)^2} \]

\[ AB = \sqrt{16 + 9} \]

\[ AB = \sqrt{25} \]

\[ AB = 5 \] The length of segment AB is 5 units.
## Area

<table>
<thead>
<tr>
<th>2-Dimensional Figure</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Triangle</strong></td>
<td>$A = \frac{1}{2}bh$</td>
<td>Find the area.</td>
</tr>
<tr>
<td><img src="image" alt="Triangle Diagram" /></td>
<td></td>
<td>The base is 5 mm and the height is 4 mm. $A = \frac{1}{2} \times 5 \times 4 = 10\text{mm}^2$</td>
</tr>
<tr>
<td><strong>Trapezoid</strong></td>
<td>$A = \frac{1}{2}h(b_1 + b_2)$</td>
<td>Find the area.</td>
</tr>
<tr>
<td><img src="image" alt="Trapezoid Diagram" /></td>
<td></td>
<td>The height is 5 ft, base$_1$ is 10 ft, and base$_2$ is 6 ft. $A = \frac{1}{2} \times 5(10 + 6) = 40\text{ft}^2$</td>
</tr>
</tbody>
</table>
Sum of Angles of a Triangle

- The sum of all of the interior angles of a triangle is 180.

  \[ \angle A + \angle B + \angle C = 180^\circ \]

- An exterior angle of a triangle is the supplement of the adjacent angle of the triangle.

  \[ \angle 1 + \angle 2 = 180^\circ \]

- An exterior angle of a triangle is equal to the sum of the two remote interior angles.

  \[ \angle 1 + \angle 3 = \angle 2 \]
Examples:

If $\overline{AC} = \overline{BC}$, and $\angle BAC = 47^\circ$, and $\angle ADB = 91^\circ$, then find $\angle DAB$.

a. 30°

b. 36°

c. 42°

d. 47°

e. 91°

$\overline{AC} = \overline{BC} \therefore \angle BAC = \angle CBA$

$\angle DAB = 180^\circ - 91^\circ - 47^\circ = 42^\circ$

If $\angle BAC = 70^\circ$, and $\angle ABC = 60^\circ$, and $\overline{CE}$ bisects $\angle BCD$, then find $\angle BCE$.

a. 55°

b. 60°

c. 65°

d. 70°

e. 75°

$\angle ACB = 180^\circ - 60^\circ - 70^\circ = 50^\circ$

$\angle BCD = 180^\circ - 50^\circ = 130^\circ$

Straight angles = 180°

To bisect means to cut in half $\therefore$

$\angle BCE = \frac{130^\circ}{2} = 65^\circ$
Special Right Triangles

45-45-90 Triangle
Length of hypotenuse is $\sqrt{2}$ times the length of a leg.

30-60-90 Triangle
Hypotenuse is twice as long as the length of the leg opposite the 30° angle. The leg opposite the 60° angle is $\sqrt{3}$ times the length of the leg opposite the 30° angle.
A right triangle has the dimensions as shown in the diagram below.

**30-60-90 Triangle**
Hypotenuse is twice as long as the length of the leg opposite the $30^\circ$ angle. The leg opposite the $60^\circ$ angle is $\sqrt{3}$ times the length of the leg opposite the $30^\circ$ angle.

What is the approximate area of the triangle?

A. 8.0 square inches
B. 11.3 square inches
C. 13.9 square inches
D. 16.0 square inches

Area (triangle) = ($b \times h$)/2
= $4(4\sqrt{3})/2$
= 13.856 $\approx$ 13.9 in$^2$

C: Correct
Pythagorean Theorem

\[ a^2 + b^2 = c^2 \]

(where \( a \) and \( b \) are the legs and \( c \) is the hypotenuse)

Examples:

1. \[ 3^2 + 4^2 = c^2 \]
   \[ 9 + 16 = 25 = c^2 \]
   so \( c = 5 \)

2. \[ 5^2 + b^2 = 13^2 \]
   \[ 25 + b^2 = 169 \]
   \[ b^2 = 144 \]
   so \( b = 12 \)
The anchoring wire of a telephone pole has snapped and needs to be replaced. The telephone pole is 30 feet tall. The anchor for the wire is 13.8 feet from the bottom of the pole.

Pythagorean Theorem
\[ a^2 + b^2 = c^2 \]
\[ 13.8^2 + 30^2 = c^2 \]
\[ 190.44 + 900 = c^2 \]
\[ c^2 = 1090.44 \]
\[ c = 33.02 \approx 35 \text{ ft} \]

Which of these is approximately the minimum length necessary for the new wire?

A. 10 ft
B. 21 ft
C. **35 ft**  \( \text{C (35 ft): Correct} \)
D. 44 ft
Congruency

- **Congruency** is used to describe the relationship between figures in geometry.
- Being congruent means having equal measurements, same shape and same size. If one figure were laid on top of the other, there would be an exact fit.
- The symbol used to show congruence is $\cong$.
- When two figures have the same shape and size, they are said to be congruent figures. Their matching parts are called corresponding parts. Congruent angles have the same measure of degrees. Congruent lines have the same measure of length.
Below are the corresponding congruent parts for the triangles pictured above.

\[ \overline{AB} \cong \overline{DE} \quad \text{and} \quad \overline{BC} \cong \overline{EF} \quad \text{and} \quad \overline{CA} \cong \overline{FD} \]

\[ \angle A \cong \angle D \quad \text{and} \quad \angle B \cong \angle E \quad \text{and} \quad \angle C \cong \angle F \]

Looking at the pictures again, notice the notations used for each congruent and corresponding pair of angles.

- the angles with the box notation are congruent right angles and are both 90, \( \angle B \cong \angle E \)
- the angles with 2 arcs are congruent and are both 60, \( \angle A \cong \angle D \)
- the angles with 1 arc are congruent and are both 30, \( \angle C \cong \angle F \)

There are other notations that show when sides are congruent and corresponding.

- the sides with 3 dashes on them are congruent to each other, \( \overline{CA} \cong \overline{FD} \)
- the sides with 2 dashes on them are congruent to each other, \( \overline{BC} \cong \overline{EF} \)
- the sides with 1 dash on them are congruent to each other, \( \overline{AB} \cong \overline{DE} \)

Since all the angles and sides are congruent, \( \triangle ABC \cong \triangle DEF \). Notice when describing triangle congruency, corresponding vertices are used.
Congruent Triangles: Triangles can be proved to be congruent by the following theorems:

1.) **Side-Side-Side** (SSS) - if three sides of a triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

2.) **Side-Angle-Side** (SAS) - if two sides of a triangle and the included angle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.

3.) **Angle-Side-Angle** (ASA) - if two angles and the included side of a triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent.

4.) **Angle-Angle-Side** (AAS) - if two angles and a non-included side of a triangle are congruent to the corresponding angles and a non-included side of another triangle, then the triangles are congruent.

5.) **Hypotenuse-Leg** (HL) - if the hypotenuse and a leg of a right triangle are congruent to the corresponding sides of another right triangle, then the triangles are congruent.
Two shapes are similar (~) if their corresponding angles are equal and their corresponding line segments are proportional. The objects have the same shape, but may not be the same size.

Example:

Question: The two triangles above are similar because their corresponding angles are congruent. What is the length of side \(x\) on the smaller triangle?

Answer: The sides of similar figures are always proportional, so

\[
\frac{12}{6} = \frac{6}{x}
\]

\[
x = 3 \text{ ft}
\]
Example:

Pretend that there is a large oak tree in your front yard that is too tall to measure. With summer storms just around the corner, your dad is concerned that during a storm the tree may fall on the house. He is not sure how tall the tree is, but he wants to know the height so that he can determine if the tree will fall on the house or fall just short of the house.

You can help your dad by using the properties of similar figures.

On a sunny day, the sun will cast proportional shadows from objects. NOTICE the corresponding parts below. The angles are congruent and the sides are proportional. Use the steps below to find the tree's height.

1. Measure the length of the shadow that the tree casts.
2. Measure the height of your dad.
3. Measure the length of the shadow that your dad casts.

By using the ratios of the measures of the tree and the measures of your dad, we can write the proportion below.

$$\frac{\text{height of the tree}}{\text{length of the shadow cast by the tree}} = \frac{\text{height of your dad}}{\text{length of the shadow cast by your dad}}$$

You don't know the height of the tree, so you will put an x in for that measurement. Now, just plug the measurements you know into the proportion.

$$\frac{x}{10} = \frac{6}{4}$$

Cross multiply to get the equation below.

$$4x = 60$$

Then solve for x.

$$x = 15 \text{ feet}$$

You have just calculated that the tree stands 15 feet tall.
The two triangles are similar triangles (have the same shape, not necessarily the same size). Similar Figures have: congruent corresponding angles and lengths of corresponding sides have the same ratio.
Similar figures are objects that have the same shape, not necessarily the same size. If two polygons are similar, then their corresponding angles are congruent (have the same measure) and the lengths of their corresponding sides have the same ratio.

The shadow cast by a one-foot ruler is 8 inches long. At the same time, the shadow cast by a pine tree is 24 feet long.

Set up a proportion:

\[
\frac{1}{2/3} = \frac{x}{24}
\]

Cross multiply:

So \(2/3x = 24\)

Solve for \(x\):

\(X = 24 \times \frac{3}{2} = 36\) ft

What is the height, in feet, of the pine tree?

A. 3 feet
B. 16 feet
C. 36 feet
D. 192 feet

C (36 ft): Correct
Examples:

Find \( x \).

a. 5 units
b. \( \frac{10}{42} \) units
c. \( \frac{21}{5} \) units
d. 6 units
e. \( \frac{10}{7} \) units

Remembe that similar triangles have proportional sides. \( \frac{\text{long side}}{\text{short side}} \) big triangle \( \frac{10}{7} \) small triangle \( \frac{6}{x} \)

Cross multiply, then divide. \( 10x = 7 \times 6 = 42 \)

\[ x = \frac{42}{10} = \frac{21}{5} \]

Find \( x \).

a. 5 units
b. \( \frac{21}{5} \) units
c. 4 units
d. \( \frac{7}{3} \) units
e. \( \frac{35}{3} \) units

Cross multiply, then divide. \( 5x = 7 \times 3 = 21 \)

\[ x = \frac{21}{5} \]
Trigonometric Ratios

There are six basic trigonometric ratios used in trigonometry: sine (sin), cosine (cos), tangent (tan), secant (sec), cosecant (csc), and cotangent (cot).

\[
\begin{align*}
sin A &= \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}} = \text{ORANGE} \\
cos A &= \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}} = \text{ALWAYS} \\
\tan A &= \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}} = \text{ORANGE} \\
csc A &= \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite}} = \text{HIPPOS} \\
sec A &= \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent}} = \text{HAVE} \\
cot A &= \frac{b}{a} = \frac{\text{adjacent}}{\text{opposite}} = \text{ANKLES}
\end{align*}
\]
Trigonometry is useful in finding the measures of missing sides and angles of right triangles. Below is a review of the basic trigonometric functions.

\[
\sin A = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

\[
\tan A = \frac{\text{opposite}}{\text{adjacent}}
\]

In this case, the reference angle is \( \angle A \). Opposite represents the side opposite of \( \angle A \) and adjacent represents the side adjacent to (beside) \( \angle A \). The hypotenuse is always the side across from the right angle.
The same triangle is listed below. But now the reference angle is $\angle B$.

\[
\sin B = \frac{\text{opposite}}{\text{hypotenuse}}
\]
\[
\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}
\]
\[
\tan B = \frac{\text{opposite}}{\text{adjacent}}
\]

Opposite represents the side opposite of $\angle B$ and adjacent represents the side adjacent to (beside) $\angle B$. **Notice that the adjacent and opposite sides change depending on which angle is the reference angle.** BUT the hypotenuse is **always** the side across from the right angle.
Example:

Phil and Tom are using an elevator to transport the straw bales into the hayloft. The bottom of the elevator is 28 feet from the barn. If the elevator makes a 30° angle with the ground, what is the distance from the ground to the bottom of the hayloft door? Round answer to the nearest whole foot.

The side adjacent to the 30° angle is 28 feet.
The question is asking for the side opposite the 30° angle.

\[
\tan A = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan 30 = \frac{x}{28}
\]

Multiply both sides by 28.

28 \tan 30 = x

On the calculator, type in \(28 \times 30 \tan =\)

\(x = 16\) feet
Example: Harrison is building a skateboard ramp in his driveway with a 6 foot board. The ramp is 2 feet high. What is the angle of elevation to the nearest tenth of a degree?

The side opposite angle \( x \) is 2

The hypotenuse is 6.

\[
\sin x = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\sin x = \frac{2}{6}
\]

To find a missing angle measure, use the inverse \( \sin^{-1} \) function on the calculator.

Type in:

\[
2 \div 6 = \text{INV} \sin \text{ Angle } x = 19.5
\]
Example: A ladder is placed against a building so that it forms a right triangle. How long of a ladder do you need to reach the top of the building that is 80 feet high?

The side opposite of the 70 angle is the height of the building (80 feet). The ladder is the hypotenuse.

Therefore the \( \sin A \) formula can be used to solve this problem because it contains the opposite and the hypotenuse.

\[
\sin A = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\sin 70 = \frac{\text{80}}{x}
\]

Multiply both sides by \( x \).

\[
x \sin 70 = 80
\]

Divide both sides by \( \sin 70 \) to solve for \( x \).

\[
x = \frac{80}{\sin 70}\]

On the calculator, type in

\[
80 ÷ 70 \sin = x = 85.13 \text{ feet}
\]

The ladder would have to be 85.13 feet long.
The Americans with Disabilities Act states that a wheelchair ramp must have a slope no greater than $\frac{1}{12}$, as shown in the diagram below.

Which of the following inequalities must be true of a ramp conforming to the Americans with Disabilities Act?

A. $\tan A \leq \frac{1}{12}$  
B. $\tan A > \frac{1}{12}$  
C. $\tan A \geq \frac{12}{1}$  
D. $\tan A = 12$

A: Correct

$\tan A = \frac{\text{opposite}}{\text{adjacent}}$

Slope must be smaller than $1/12$
A yacht is anchored 90 feet offshore from the base of a lighthouse. The angle of elevation from the boat to the top of the lighthouse is 26 degrees. The distance between the yacht and the top of the lighthouse is about 100 feet.

Which of these is nearest to the height of the lighthouse?

A. 25 feet  
B. 45 feet  
C. 110 feet  
D. 135 feet

B (45 feet): Correct
1. Parts of a circle
2. Circumference
3. Area
Circles

• A **circle** is a set of points that are all the same distance from a specific point. That point is called the **center**. There are special terms that apply to circles. The **radius** is a line segment from the center of the circle to a point on the circle. The **diameter** is a line segment that passes through the center of the circle with endpoints on the circle.

• **Circumference** - the distance around a circle
  
  \[ C = 2\pi r \text{ or } C = d\pi \]

• **Area** of a circle = \( \pi r^2 \) (*area will always be in square units*)
A circle is a set of points that are all the same distance from a point. That point is called the center. A circle is named by its center. The circle to the right is called circle A or \( oA \). If a line segment is rotated about point A, then the complete rotation is 360°.

<table>
<thead>
<tr>
<th>Circle Parts</th>
<th>Representation</th>
<th>Angle Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>An <strong>arc</strong> is a portion of the circle.</td>
<td><img src="https://via.placeholder.com/150" alt="Arc" /></td>
<td>The length of an arc = ( \frac{m}{360} \times 2\pi r ), where ( m ) is the degree measure of the central angle.</td>
</tr>
<tr>
<td>A <strong>central angle</strong> is an angle with its vertex at the center and sides that are radii (plural for radius).</td>
<td><img src="https://via.placeholder.com/150" alt="Central Angle" /></td>
<td>Central angles are measured by their corresponding arcs. ( \hat{\angle} XYZ = \hat{\angle} XZ )</td>
</tr>
<tr>
<td>An <strong>inscribed angle</strong> is an angle with its vertex on the circle and sides that are chords.</td>
<td><img src="https://via.placeholder.com/150" alt="Inscribed Angle" /></td>
<td>Inscribed angles are measured by their corresponding arcs. ( \hat{\angle} ABC = \frac{1}{2} (\hat{\angle} AC) )</td>
</tr>
</tbody>
</table>
Degree Measure of an Arc

• The degree measure of a minor arc is equal to the degree measure of its central angle. The degree measure of a major arc is equal to 360 minus the degree of its central angle. The degree measure of a semicircle is 180.

Length of an Arc

• Two arcs may have the same degree measure, but not be congruent. When you are finding the length of an arc, you are really finding part of the circumference of the circle. The length of an arc is \( \frac{m\pi d}{180} \), where \( m \) is the degree measure of the central angle. Two arcs are congruent if they have the same length and are parts of the same circle or congruent circles.
<table>
<thead>
<tr>
<th>Definition</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A tangent</strong> is a line that intersects the circle at one point.</td>
<td>![Tangent Diagram]</td>
</tr>
<tr>
<td><strong>A secant</strong> is a line that passes through two points on the circle.</td>
<td>![Secant Diagram]</td>
</tr>
<tr>
<td><strong>A sector</strong> is a region enclosed by two radii and an arc.</td>
<td>![Sector Diagram]</td>
</tr>
<tr>
<td><strong>A segment</strong> is a region enclosed by a chord and an arc.</td>
<td>![Segment Diagram]</td>
</tr>
</tbody>
</table>

An **arc** is a portion of the circle. Three types of arcs are **minor arcs**, **semicircles**, and **major arcs**. Minor arcs have central angles $< 180$. Semicircles have central angles $= 180$. Major arcs have central angles $> 180$.

NOTE: The diameter (example: XY) divides the circle into two arcs called semi-circles.

**A central angle** is an angle with its vertex at the center and sides that are radii.

**An inscribed angle** is an angle with its vertex on the circle and sides that are chords.
## Circumference & Area

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Circumference of a Circle</strong></td>
<td>$C = 2\pi r$ or $C = \pi d$</td>
<td>Find the circumference of a circle whose radius is 5 cm. Round to the nearest tenth. $C = 2 \cdot \pi \cdot 5 = 31.4$ cm</td>
</tr>
<tr>
<td><strong>Area of a Circle</strong></td>
<td>$A = \pi r^2$</td>
<td>Find the area of the given circle to the nearest hundredth. $A = \pi \cdot 7^2 = 153.94$ m$^2$</td>
</tr>
</tbody>
</table>

**If the diameter of Circle A is 20 units, what area remains if Circle B is removed from Circle A?**

- a. $75\pi$ units$^2$
- b. $100\pi$ units$^2$
- c. 50 units$^2$
- d. 75 units$^2$
- e. $25\pi$ units$^2$

**Diameter for Circle A = 20 units**

**Radius for Circle A = 10 units**

**Radius for Circle B = 5 units**

**Area for Circle A = $\pi(10^2) = 100\pi$**

**Area for Circle B = $\pi(5^2) = 25\pi$**

**Area for Circle A – Area for Circle B = $75\pi$**
In the diagram below, line segment AT is a diameter of the circle with center O.

What is the area of the shaded part of the circle?

A. 18.3 cm²
B. 42.3 cm²
C. **145.6 cm²**
D. 194.1 cm²

**C: Correct**

Area of circle = \( \pi r^2 \)

= \( 3.14 \times 8 \times 8 \) = 200.96

Area of triangle = \( (b \times h)/2 \)

= \( (8 \times 8\sqrt{3})/2 \) = 55.42

Shaded = 200.96 – 55.42 = **145.54**

See formula sheet for special right triangles
Examples:

_0 is the center of the circle; the radius of this circle is 7 units. \( BC \) is 8 units. \( OA \) bisects \( BC \). What is \( OD \)?_

- a. 7 units
- b. 8 units
- c. 10 units
- d. \( \sqrt{65} \) units
- e. \( \sqrt{33} \) units

If \( OA = 9 \) units, and \( OB = 6 \) units, then find \( DB \).

- a. 9 units
- b. 6 units
- c. 15 units
- d. 3 units
- e. 5 units

If \( \angle OBA = 33^\circ \), then find \( \angle AOB \).

- a. 33°
- b. 147°
- c. 157°
- d. 114°
- e. 120°
Examples:

If $\angle BOA = 120^\circ$, then find $\angle ABO$

a. $120^\circ$

b. $60^\circ$

c. $80^\circ$

d. $15^\circ$

e. $30^\circ$

$\angle BOA = 120^\circ$; this means that the sum of the remaining two angles equals $60^\circ$. The other two angles must be equal, so $\angle ABO = \frac{60^\circ}{2} = 30^\circ$

If the circumference of a circle is $14\pi$ units, what is the radius?

a. 7 units

$Circumference = \pi d = 2\pi r$

b. 14 units

$2\pi r = 14\pi$

c. 28 units

$r = \frac{14\pi}{2\pi} = 7$

d. $7\pi$ units

e. 12 units

What is the area of a circle inscribed with a square whose sides are 16 units long each?

a. 64 units$^2$

b. 8 units$^2$

c. $8\pi$ units$^2$

d. $64\pi$ units$^2$

e. $4\pi$ units$^2$

Diameter $= 2 \times$ radius

Area $= \pi r^2 = \pi (x^2) = 64\pi$

If the area of a circle is $25\pi$ units$^2$, then what is the circle’s diameter?

a. 25 units

Area $= \pi r^2 = 25\pi$

Diameter $= 2r$

c. $\sqrt{5}$ units

$r^2 = \frac{25\pi}{\pi}$

Diameter $= 2 \times 5$

d. 50 units

$r = \sqrt{25} = 5$

Diameter $= 10$

e. 10 units
Angles

1. Supplementary
2. Complementary
3. Adjacent
4. Vertical
5. Parallel Lines
6. Perpendicular Lines
1. Two angles are **supplementary** if the sum of their measures equals 180°.
   \[ m\angle A + m\angle B = 180^\circ \]

2. Two angles are **complementary** if the sum of their measures equals 90°.
   \[ m\angle A + m\angle B = 90^\circ \]

**Examples:**

**Question:** \( \angle A \) is **supplementary** to \( \angle B \). If \( \angle A \) is 45°, what is the measure of \( \angle B \)?

**Answer:** 135°; because \( 180^\circ - 45^\circ = 135^\circ \)

**Question:** \( \angle A \) is **complementary** to \( \angle B \). If \( \angle A \) is 25°, what is the measure of \( \angle B \)?

**Answer:** 65°; because \( 90^\circ - 25^\circ = 65^\circ \)
Vertical angles are congruent angles that are formed by intersecting lines. They appear opposite of each other when two lines intersect.

∠1 and ∠4 are vertical angles. ∠1 ≅ ∠4

∠2 and ∠3 are vertical angles. ∠2 ≅ ∠3

Question: If ∠A is 130°, what is the measure of ∠B?

Answer: 130°; because vertical angles are always congruent
**Parallel and Perpendicular Lines**

**Parallel lines** are lines that are in the same plane and do NOT intersect.

**Example:**
As you can see in the picture above, no matter how far out lines \( L1 \) and \( L2 \) are extended, they will NEVER intersect. The lines above are parallel. Parallel lines have the same slope.

**Perpendicular lines** are lines that are in the same plane and intersect to form a **right angle**. A **right angle** is perfectly square and measures 90°.

**Example:**
As you can see in the picture above, the lines \( L1 \) and \( L2 \) intersect to form a **right angle**. They are perpendicular. Perpendicular lines have the negative reciprocal slope.

If two parallel lines are cut by a **transversal** (a line that intersects two or more lines at different points) the two alternate interior angles are congruent. See figure below.

**Example:**
In the picture above, lines \( L1 \) and \( L2 \) are parallel. Angles \( A \) and \( D \) are congruent. Angles \( B \) and \( C \) are congruent.
When **parallel lines** are intersected by a **transversal**, eight angles are formed that have some unique relationships.

There are four pairs of **vertical angles**. \[\angle 1 \cong \angle 4, \quad \angle 2 \cong \angle 3, \quad \angle 5 \cong \angle 8, \quad \angle 6 \cong \angle 7\]

Notice that if you slide line B onto line A, the overlapping angles become corresponding parts known as corresponding angles. These angles are congruent. There are four pairs of **corresponding angles**. \[\angle 1 \cong \angle 5, \quad \angle 2 \cong \angle 6, \quad \angle 3 \cong \angle 7, \quad \angle 4 \cong \angle 8\]

Notice the notations used in the diagram below to show the congruent pairs of **corresponding angles**.
<table>
<thead>
<tr>
<th>Term</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pairs of alternate interior angles</strong> are congruent. They are on opposite sides of the transversal and on the inside of the parallel lines.</td>
<td>$\angle 3 = \angle 6$&lt;br&gt;$\angle 4 = \angle 5$</td>
</tr>
<tr>
<td><strong>Pairs of alternate exterior angles</strong> are congruent. They are on opposite sides of the transversal and on the outside of the parallel lines.</td>
<td>$\angle 1 = \angle 8$&lt;br&gt;$\angle 2 = \angle 7$</td>
</tr>
<tr>
<td><strong>Supplementary angles</strong> are angles that add up to 180 degrees. There are many supplementary angles when two parallel lines are cut by a transversal.</td>
<td>Angles that are on the same side of a line and are adjacent like $\angle 1$ and $\angle 2$.</td>
</tr>
<tr>
<td><strong>Pairs of consecutive interior angles</strong> are supplementary. They are on the same side of the transversal and on the inside of the parallel lines. Consecutive interior angles are also known as same-side interior angles.</td>
<td>$\angle 3 + \angle 5 = 180$&lt;br&gt;$\angle 4 + \angle 6 = 180$</td>
</tr>
</tbody>
</table>
In which figure is the measure of $\angle 1$ not equal to $60^\circ$?

A. $70^\circ$  
A: Correct  
Sum of the degrees in a straight line = 180

B. $30^\circ$  
Sum of the degrees in a right angle = 90

C. $120^\circ$  
Sum of the degrees in a straight line = 180

D. $120^\circ$  
Corresponding angles are congruent.  
Sum of the degrees in a straight line = 180
When a marble hits a wall, it bounces off the wall at the same angle it hits the wall.

If a marble hits a wall at a 22 degree angle, what is the measure of the angle between the two paths of the marble?

A. 44°  
B. 68°  
C. 136°  
D. 158°

C: Correct

22 + ? + 22 = 180°
So 180 − 44 = 136°
In which figure is the measure of $\angle 1$ equal to 45°?

A. \[180 - 45 - 45 = 90^\circ\]

B. \[180 - 145 = 35^\circ\]

C. \[180 - 145 = 35^\circ\]

D. Correct

The ray divides a right angle into two angles whose total measure is 90°.
Alternate interior angles are congruent.

\( \angle 5 \cong \angle 7 \) because they are alternate interior angles.

Perpendicular angles intersect at 90 degree angles.

When framing in a wall, carpenters make sure that all vertical studs are perpendicular to the floor and ceiling. They sometimes add a diagonal brace for added support during construction (as shown in the drawing).

When the vertical studs are perpendicular to the floor, which pair of angles will always be congruent?

A. \( \angle 1 \) and \( \angle 2 \)  Supplementary (add up to 180°)
B. \( \angle 3 \) and \( \angle 6 \)  Supplementary (add up to 180°)
C. \( \angle 5 \) and \( \angle 7 \)  \( \angle 5 \) and \( \angle 7 \) are congruent: Correct
D. \( \angle 6 \) and \( \angle 9 \)  Supplementary (add up to 180°)
A worker painted stripes for spaces in a parking lot. The worker first painted a center stripe that marked the front of the parking spaces. Then he painted parallel stripes marking the sides.

Which angles will be congruent to angle 1 if all the side stripes are parallel?

A. \( \angle 2 \) and \( \angle 3 \)
B. \( \angle 2 \) and \( \angle 5 \)
C. \( \angle 3 \) and \( \angle 5 \)
D. \( \angle 4 \) and \( \angle 5 \)

C: Correct
Examples:

*If lines $a$ and $b$ are parallel, which angles have the same measure as $\angle 1$?*

- a. $\angle 2$, $\angle 3$, and $\angle 6$
- b. $\angle 5$, $\angle 6$, and $\angle 8$
- c. $\angle 2$, $\angle 3$, and $\angle 4$
- d. All of the angles
- e. $\angle 6$, $\angle 5$, and $\angle 8$

Remember that in the case of parallel lines, angles across from one another are equal, and two angles next to each other add up to $180^\circ$.

*If lines $a$ and $b$ are parallel, and $\angle 1 = 135^\circ$, then find $\angle 6$.*

- a. $135^\circ$
- b. $45^\circ$
- c. $180^\circ$
- d. $35^\circ$
- e. $100^\circ$

Remember that in the case of parallel lines, angles across from one another are equal, and two angles next to each other add up to $180^\circ$. 
Rectangles

1. Types of Quadrilaterals
2. Perimeter
3. Area
# Quadrilaterals

<table>
<thead>
<tr>
<th>Quadrilateral (A quadrilateral is a polygon with 4 sides and 4 angles)</th>
<th>Representation</th>
</tr>
</thead>
</table>
| **Parallelogram** is a 4-sided shape. Opposite sides are parallel.  
Opposite sides are congruent.  
Opposite angles are congruent. | ![Parallelogram](image) |
| **Rectangle** is a parallelogram with four right angles. | ![Rectangle](image) |
| **Rhombus** is a parallelogram with four equal sides. | ![Rhombus](image) |
| **Square** is a parallelogram with four right angles and four equal sides. A square is a special kind of rectangle and rhombus. | ![Square](image) |
| **Trapezoid** is a 4-sided quadrilateral that has only one set of parallel lines. | ![Trapezoid](image) |
PROPERTIES OF QUADRILATERALS

- The sum of the angles in any quadrilateral is equal to \(360^\circ\).

**PARALLELOGRAM** - a quadrilateral in which both pairs of opposite sides are parallel
- Opposite sides are congruent.
- Opposite angles are congruent.
- Any pair of consecutive angles are supplementary.
- Diagonals bisect each other.
- Area = \(bh\) (base times height)

**RECTANGLE** - a parallelogram in which at least one angle is a right angle.
- Opposite sides are parallel and congruent.
- All angles are right angles.
- Any pair of consecutive angles are supplementary.
- Diagonals are congruent and bisect each other.
- Area = \(lw\) (length times width)

**RHOMBUS** - a parallelogram in which at least two consecutive sides are congruent.
- Opposite sides are parallel.
- All sides are congruent.
- Opposite angles are congruent.
- Any pair of consecutive angles are supplementary.
- Diagonals are perpendicular bisectors of each other.
- Diagonals bisect the angles of a rhombus.
- Area = \(bh\) (base times height)
**SQUARE** - a parallelogram that is both a rectangle and a rhombus.
- Opposite sides are parallel and congruent.
- All angles are right angles.
- Any pair of consecutive angles are supplementary.
- Diagonals are congruent and perpendicular bisectors of each other.
- Diagonals bisect the angles of a square.
- Diagonals form four isosceles right triangles (45°-45°-90° triangles).
- Area = lw (length times width) **OR** Area = $s^2$ (side squared)

**KITE** - a quadrilateral with two distinct pairs of consecutive congruent sides.
- One diagonal is the perpendicular bisector of the other diagonal.
- $A = \frac{d_1 + d_2}{2}$ (sum of diagonals divided by 2)

**TRAPEZOID** - a quadrilateral with exactly one pair of parallel sides (bases).
- Area = $\frac{h(b_1 + b_2)}{2}$ (sum of the bases times the height, divided by 2)

**ISOSCELES TRAPEZOID** - a trapezoid in which the pair of non-parallel sides are congruent (legs).
- Lower base angles are congruent.
- Upper base angles are congruent.
- Any lower base angle is supplementary to any upper base angle.
- Diagonals are congruent.
- Area = $\frac{h(b_1 + b_2)}{2}$
**Area**

Area is the number of square units enclosed by a 2-dimensional figure. Area is measured in **square units**. Remember to refer to the Mathematics Reference Sheet to find the area formula.

<table>
<thead>
<tr>
<th>2-Dimensional Figure</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
<td>A = lw</td>
<td><strong>Find the area.</strong></td>
</tr>
<tr>
<td></td>
<td>NOTE: since a square is a rectangle it has the same area formula with the length and width being the same measure</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The length is 8 cm and the width is 3 cm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 8 \times 3 = 24 \text{ cm}^2</td>
</tr>
<tr>
<td></td>
<td><strong>Find the area.</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The base is 11 cm and the height is 6 cm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 6 \times 11 = 66 \text{ cm}^2</td>
</tr>
</tbody>
</table>

**Parallelogram**

NOTE: The height is perpendicular to the base.

<table>
<thead>
<tr>
<th>2-Dimensional Figure</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A = bh</td>
<td><strong>Find the area.</strong></td>
</tr>
<tr>
<td></td>
<td>NOTE: since a rhombus is a parallelogram it has the same area formula</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The base is 11 cm and the height is 6 cm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 6 \times 11 = 66 \text{ cm}^2</td>
</tr>
</tbody>
</table>
### Polygons

<table>
<thead>
<tr>
<th>Definition</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A <strong>triangle</strong> is a 3-sided polygon.</td>
<td>![Triangle]</td>
</tr>
<tr>
<td>A <strong>quadrilateral</strong> is a 4-sided polygon.</td>
<td>![Quadrilateral]</td>
</tr>
<tr>
<td>A <strong>pentagon</strong> is a 5-sided polygon.</td>
<td>![Pentagon]</td>
</tr>
<tr>
<td>A <strong>hexagon</strong> is a 6-sided polygon.</td>
<td>![Hexagon]</td>
</tr>
<tr>
<td>A <strong>heptagon</strong> is a 7-sided polygon.</td>
<td>![Heptagon]</td>
</tr>
<tr>
<td>An <strong>octagon</strong> is an 8-sided polygon.</td>
<td>![Octagon]</td>
</tr>
<tr>
<td>A <strong>nonagon</strong> is a 9-sided polygon.</td>
<td>![Nonagon]</td>
</tr>
<tr>
<td>A <strong>decagon</strong> is a 10-sided polygon.</td>
<td>![Decagon]</td>
</tr>
<tr>
<td>A <strong>convex polygon</strong> does not have a vertex in the interior of the polygon.</td>
<td>![Convex Polygon]</td>
</tr>
<tr>
<td>A <strong>concave polygon</strong> has at least one vertex in the interior of the polygon. (Think of it as having a dent.)</td>
<td>![Concave Polygon]</td>
</tr>
</tbody>
</table>
Geometry: Interior Angles

The sum of interior angles $I$ in a polygon with $n$ sides is given by the formula

$$I = (n-2) \times 180°$$

Exterior Angles

An exterior angle of a convex polygon is the angle formed between a side of a polygon and the extension of an adjacent side. Since there are two directions in which a side can be extended, there are two exterior angles at each vertex. However, since corresponding angles are opposite, they are also equal. The sum of exterior angles in a convex polygon is equal to $360°$, since this corresponds to one complete rotation of the polygon.
# Interior and Exterior Angles of Polygons

<table>
<thead>
<tr>
<th>Shape</th>
<th>Number of sides n</th>
<th>Interior angle sum 180 (n - 2)</th>
<th>To find each interior angle, divide the sum by the number of sides 180(n-2) ÷ n</th>
<th>Exterior angle sum 360</th>
<th>To find each exterior angle, divide the sum by the number of sides 360 ÷ n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>3</td>
<td>180</td>
<td>60</td>
<td>360</td>
<td>120</td>
</tr>
<tr>
<td>Square</td>
<td>4</td>
<td>180</td>
<td>60</td>
<td>360</td>
<td>90</td>
</tr>
<tr>
<td>Regular Pentagon</td>
<td>5</td>
<td>540</td>
<td>108</td>
<td>360</td>
<td>72</td>
</tr>
<tr>
<td>Regular Hexagon</td>
<td>6</td>
<td>720</td>
<td>120</td>
<td>360</td>
<td>60</td>
</tr>
<tr>
<td>Regular Heptagon</td>
<td>7</td>
<td>900</td>
<td>≈128.6</td>
<td>360</td>
<td>≈51.4</td>
</tr>
<tr>
<td>Regular Octagon</td>
<td>8</td>
<td>1080</td>
<td>135</td>
<td>360</td>
<td>45</td>
</tr>
<tr>
<td>Regular Nonagon</td>
<td>9</td>
<td>1260</td>
<td>140</td>
<td>360</td>
<td>40</td>
</tr>
<tr>
<td>Regular Decagon</td>
<td>10</td>
<td>1440</td>
<td>144</td>
<td>360</td>
<td>36</td>
</tr>
<tr>
<td>Regular n-gon</td>
<td>n</td>
<td>180(n × 2)</td>
<td>180(n-2) ÷ n</td>
<td>360</td>
<td>360 ÷ n</td>
</tr>
</tbody>
</table>
Example: Find x.

- The sum of the interior angles is $180(n - 2)$ and $n = 4$ since the figure has four sides.
  
  $180(4 - 2) = 180(2) = 360$.

- **The sum of the interior angles = 360.** Add the angle measure you already know, $90 + 90 + 120 = 300$.

- Subtract the known angle measures from the sum of all the angles, $360 - 300 = 60$. $x = 60$
Examples:

1. If the length of a rectangle is 12 units and its width is 9 units, what is the length of its diagonal?
   a. 10 units
   b. 12 units
   c. 13 units
   d. **15 units**
   e. 21 units
   \[ c = \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = 15 \]

2. If \( AC = 10 \) units and \( AB = 4 \) units, then what is \( AD \)?
   a. 6 units
   b. **\( 2\sqrt{21} \) units**
   c. \( \sqrt{89} \) units
   d. 10 units
   e. 40 units
   \[ AD = BC \]
   \[ BC = \sqrt{10^2 - 4^2} = \sqrt{84} \]
   \[ \sqrt{84} = \sqrt{4 \times 21} = \sqrt{4} \times \sqrt{21} = 2\sqrt{21} \]

3. If the area of the square is 100 units\(^2\) and \( CE = 12 \) units, what is \( BE \)?
   a. **\( 2\sqrt{11} \) units**
   b. \( \sqrt{83} \) units
   c. 10 units
   d. 12 units
   e. 14 units
   If the area of the square is 100, then each side must be 10.
   \[ BE = \sqrt{12^2 - 10^2} = \sqrt{144 - 100} \]
   \[ \sqrt{44} = 2\sqrt{11} \]
Examples:

What is the area of this figure?

a. 10 units$^2$

b. 20 units$^2$

c. **30 units$^2$**

d. 40 units$^2$

e. 50 units$^2$

This figure is a trapezoid.

Area = $\frac{1}{2} h(B + b) = \frac{1}{2} (3)(12 + 8)$

$= \frac{1}{2} (3)(20) = \frac{1}{2} (60) = 30$

If the length and the width of a rectangle are tripled, by how many times does the area increase?

a. 3

Area for Rectangle 1 = $l \times w$

b. 6

Area for Rectangle 2 = $3l \times 3w = 9lw$

c. 27

d. **9**

e. 12

If a rectangular room, measuring 12 feet X 15 feet, is to be tiled with tiles that have 9 inch sides, how many tiles are needed to complete this room?

a. 50 tiles

b. 100 tiles

c. 150 tiles

d. 200 tiles

e. **320 tiles**

First, make all units the same; usually the smallest unit is the preferred unit. Convert the 12 feet $\times$ 15 feet room to

144 inches $\times$ 180 inches; one foot = 12 inches,

Area of room = 144 inches $\times$ 180 inches = 25,920 inches$^2$

Area of tile = 9 inches $\times$ 9 inches = 81 inches$^2$

$\frac{25920 \text{ inches}^2}{81 \text{ inches}^2} = 320 \text{ tiles}$
Three-Dimensional Concepts

1. Polyhedrons
   - Prisms
   - Pyramids

2. Other 3-D Objects
   - Cylinder
   - Cone
   - Sphere

3. Surface area

4. Volume
**Prisms** have 2 parallel and congruent bases (top and bottom) that can be any polygon.

The faces are the sides and are **always** rectangular for prisms.

Some examples are shown to the right.

<table>
<thead>
<tr>
<th>Prisms</th>
<th>Representation</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular prism</td>
<td><img src="image" alt="Triangular prism" /></td>
<td><img src="image" alt="Triangular prism net" /></td>
</tr>
<tr>
<td>(bases are triangles)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular prism</td>
<td><img src="image" alt="Rectangular prism" /></td>
<td><img src="image" alt="Rectangular prism net" /></td>
</tr>
<tr>
<td>(bases are rectangles)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cube</td>
<td><img src="image" alt="Cube" /></td>
<td><img src="image" alt="Cube net" /></td>
</tr>
<tr>
<td>(all sides are squares)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Pyramids have only one base (bottom) that can be any polygon.

The faces are the sides and are always triangles for pyramids.

Some examples are shown to the right.

<table>
<thead>
<tr>
<th>Pyramids</th>
<th>Representation</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular pyramid</td>
<td><img src="image" alt="Triangular pyramid" /></td>
<td><img src="image" alt="Triangular pyramid net" /></td>
</tr>
<tr>
<td>Square pyramid</td>
<td><img src="image" alt="Square pyramid" /></td>
<td><img src="image" alt="Square pyramid net" /></td>
</tr>
</tbody>
</table>

**Pyramids**
## Other 3-Dimensional Objects

<table>
<thead>
<tr>
<th>Solid (using circles)</th>
<th>Representation</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cylinders</strong> have 2 parallel circular bases.</td>
<td><img src="image1" alt="Cylinder Diagram" /></td>
<td><img src="image2" alt="Cylinder Net Diagram" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The circumference of the circle must equal the length of the rectangle.</td>
</tr>
<tr>
<td><strong>Cones</strong> have only one circular base.</td>
<td><img src="image3" alt="Cone Diagram" /></td>
<td><img src="image4" alt="Cone Net Diagram" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The circumference of the circle must equal the length of the arc of the given sector.</td>
</tr>
<tr>
<td><strong>Spheres</strong> are a set of points that are equal distance from the center.</td>
<td><img src="image5" alt="Sphere Diagram" /></td>
<td>No net</td>
</tr>
</tbody>
</table>

Note: Solids that deal with circles are **not-polyhedrons**.
Surface Area – outside of the object – always measured in square units

SA (cube) = 6s^2
SA (rectangular prism) = 2lw + 2wh + 2lh
SA (cone) = \pi r^2 + \pi rs
SA (cylinder) = 2\pi r^2 + 2\pi rh
SA (pyramid) = B + \frac{1}{2} sp \text{ where } B \text{ is the area of the base and } p \text{ is the perimeter of the base}
SA (sphere) = 4\pi r^2

• Surface Area – find the area of each side, then add the them together (answer will be in square units)
Surface Area

<table>
<thead>
<tr>
<th>3-D Object</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Prism or Square Prism:</td>
<td>Find the surface area of the box below.</td>
</tr>
</tbody>
</table>

NOTE: For surface area, think of wrapping paper covering a birthday present or the amount of cardboard used to make a box.

Each side is rectangular. Area of a rectangle = lw

There are six sides and opposite sides are the same.

Area of front = 8 \times 14 = 112 \text{ cm}^2
Area of back = 8 \times 14 = 112 \text{ cm}^2
Area of top = 14 \times 5 = 70 \text{ cm}^2
Area of bottom = 14 \times 5 = 70 \text{ cm}^2
Area of right side = 8 \times 5 = 40 \text{ cm}^2
Area of left side = 8 \times 5 = 40 \text{ cm}^2
Add them together to find the surface area of this prism.

\[ \text{SA} = 112 + 112 + 70 + 70 + 40 + 40 = 444 \text{ cm}^2 \]
### Surface Area

<table>
<thead>
<tr>
<th>3-D Object</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cylinder:</strong></td>
<td><img src="image" alt="Cylinder Diagram" /></td>
</tr>
<tr>
<td>This shape can be broken into two circles and a rectangle.</td>
<td>Find the surface area of the cylinder below to the nearest hundredth.</td>
</tr>
<tr>
<td>top</td>
<td>There is a top circle and a bottom circle that are equal.</td>
</tr>
<tr>
<td>side</td>
<td>Area of a circle = $\pi r^2$</td>
</tr>
<tr>
<td>bottom</td>
<td>2 circle areas = $2 \cdot \pi \cdot r^2 = 2 \cdot \pi \cdot 2^2 = 25.13 \text{ cm}^2$</td>
</tr>
</tbody>
</table>

NOTE: For surface area, think of taking the label off of a soup can, and then include the top & bottom of the can.

The side is a rectangle. Its area formula is based on the length (circumference of the circle) times the width (height of the cylinder).

Rectangle area = $2\pi h$

Rectangle area = $2 \cdot \pi \cdot 2 \cdot 5 = 62.83 \text{ cm}^2$

Add them together to find the surface area of a cylinder.

$SA = 25.13 + 62.83 = 87.96 \text{ cm}^2$
<table>
<thead>
<tr>
<th>3-D Object</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism:</td>
<td>Find the surface area of the right triangular prism.</td>
</tr>
<tr>
<td></td>
<td>The top and bottom are triangles.</td>
</tr>
<tr>
<td></td>
<td>Area of a triangle = ( \frac{1}{2} )bh</td>
</tr>
<tr>
<td></td>
<td>Area of top = ( \frac{1}{2} ) 3 4 = 6 cm(^2)</td>
</tr>
<tr>
<td></td>
<td>Area of bottom = ( \frac{1}{2} ) 3 4 = 6 cm(^2)</td>
</tr>
<tr>
<td></td>
<td>The sides are all rectangles. Area of a rectangle = lw</td>
</tr>
<tr>
<td></td>
<td>Area of back right rectangle = 3 9 = 27 cm(^2)</td>
</tr>
<tr>
<td></td>
<td>Area of back left rectangle = 4 9 = 36 cm(^2)</td>
</tr>
<tr>
<td></td>
<td>Area of front rectangle = 5 9 = 45 cm(^2)</td>
</tr>
<tr>
<td></td>
<td>Add them together to find the surface area of this prism.</td>
</tr>
<tr>
<td></td>
<td>SA = 6 + 6 + 27 + 36 + 45 = 120 cm(^2)</td>
</tr>
<tr>
<td>3-D Object</td>
<td>Example</td>
</tr>
<tr>
<td>------------</td>
<td>---------</td>
</tr>
<tr>
<td>Pyramid: The object will vary depending on the shape of the base.</td>
<td>Find the surface area of the pyramid below.</td>
</tr>
</tbody>
</table>

**Pyramid:**

The bottom is a square. Area of a square = \( lw \)

Area of bottom = \( 6 \times 6 = 36 \text{ cm}^2 \)

The four sides are equal triangles. Triangle area = \( \frac{1}{2}bh \)

4 triangle areas = \( 4 \left( \frac{1}{2} \times 6 \times 8.5 \right) = 102 \text{ cm}^2 \)

Add them together to find the surface area of this pyramid.

\( SA = 36 + 102 = 138 \text{ cm}^2 \)
Volume

Volume – the measure of the interior of a figure (how much it will hold) - always measured in cubic units

\[ V \text{ (cube)} = s^3 \]
\[ V \text{ (rectangular prism)} = lwh \]
\[ V \text{ (cone)} = \frac{1}{3}\pi r^2 h \]
\[ V \text{ (cylinder)} = \pi r^2 h \]
\[ V \text{ (pyramid)} = \frac{1}{3} Bh \quad \text{where } B \text{ is the area of the base} \]
\[ V \text{ (sphere)} = \frac{1}{3}\pi r^3 \]

Volume \((V)\) is the amount of space inside a 3-dimensional object. It represents the number of unit cubes that will fill the object. Volume is measured in cubic units.

Remember, you do not have to memorize these volume formulas. When taking the OGT test, you can refer to the Mathematics Reference Sheet for the volume formulas.
## Volume

<table>
<thead>
<tr>
<th>3-D Object</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangular Prism:</strong></td>
<td>$V = lwh$</td>
<td>Find the volume of a rectangular prism that is 75 feet long, 30 feet wide and 5 feet deep? $V = 75 \times 30 \times 5 = 11,250 \text{ ft}^3$</td>
</tr>
<tr>
<td><img src="image" alt="Rectangular Prism Diagram" /></td>
<td>NOTE: The volume is the area of the base $(lw)$ times the height.</td>
<td></td>
</tr>
<tr>
<td><strong>Cylinder:</strong></td>
<td>$V = \pi r^2 h$</td>
<td>Find the volume of the cylinder to the nearest tenth. $V = \pi \times (3^2) \times 8 = 226.2 \text{ cm}^3$</td>
</tr>
<tr>
<td><img src="image" alt="Cylinder Diagram" /></td>
<td>NOTE: The volume is the area of the base $(\pi r^2 )$ times the height.</td>
<td></td>
</tr>
<tr>
<td><strong>Cone:</strong></td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>Find the volume of the cone to the nearest hundredth. $V = \frac{1}{3} \pi \times (2^2) \times 7 = 29.32 \text{ cm}^3$</td>
</tr>
<tr>
<td><img src="image" alt="Cone Diagram" /></td>
<td>NOTE: This formula is $\frac{1}{3}$ the volume of a cylinder formula.</td>
<td></td>
</tr>
<tr>
<td>3-D Object</td>
<td>Formula</td>
<td>Example</td>
</tr>
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<td>------------</td>
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<tr>
<td><strong>Right Prism:</strong></td>
<td>$V = Bh$, where $B$ is the area of the base&lt;br&gt;&lt;br&gt;Capital $B$ is the base area.&lt;br&gt;&lt;br&gt;Lowercase $b$ is the base length for a polygon.</td>
<td>Find the volume of the triangular prism. 3in&lt;br&gt;&lt;br&gt;The base is a triangle. The area of a triangle is $\frac{1}{2}bh$. $B = \frac{1}{2} \times 3 \times 3$&lt;br&gt;$V = \frac{1}{2} \times 3 \times 3 \times 9 = 40.5$ in$^3$</td>
</tr>
<tr>
<td><strong>Pyramid:</strong></td>
<td>$V = \frac{1}{3} Bh$, where $B$ is the area of the base and $h$ is the height (from the vertex to the base)</td>
<td>Find the volume of the pyramid. 8 cm&lt;br&gt;&lt;br&gt;The base is a rectangle. The area of a rectangle is $lw$. $B = 6 \times 7$&lt;br&gt;$V = \frac{1}{3} \times 6 \times 7 \times 8 = 112$ cm$^3$</td>
</tr>
</tbody>
</table>

**Volume**
## Volume

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of a Sphere</td>
<td>$V = \frac{4}{3} \pi r^3$</td>
<td>Find the volume of a sphere whose radius is 2 cm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V = \frac{4}{3} \pi \times 2^3 = 33.5 \text{ cm}^3$</td>
</tr>
</tbody>
</table>
Example: Estimate how many miniature candy bars can fit into a square box that has 8 inches sides. The dimensions of the miniature candy bar are shown below.

To estimate an answer, find the volume of the box and the approximate volume of each bar.

Volume = length × width × height

The volume of the box is $8 \times 8 \times 8 = 512$ inches$^3$.

The volume of a miniature candy bar can be estimated by rounding the values first. Round the length from $2\frac{1}{4}$ to 2 inches. The width is 2 inch. Round the height from $\frac{3}{4}$ to 1 inch.

The volume of a miniature candy bar is approximately $2 \times 2 \times 1 = 4$.

Now divide the box volume by the candy bar volume. $512 \div 4 = 128$

Approximately 128 miniature candy bars will fit into the square box.
Madison is filling balloons with helium. When full, the balloons are nearly spherical in shape with a diameter of 12 inches.

What is the approximate volume of each balloon when it is full?

A. 115 cubic inches
B. 450 cubic inches
C. 680 cubic inches
D. 900 cubic inches

If diameter = 12 then radius = 6.

\[ V = \frac{4}{3} \pi r^3 \]
\[ 4/3 \times (3.14)(6)(6)(6) = 904.32 \]

D: Correct
Mr. Anderson has machinery on his farm that allows him to bale hay two different ways: “round” and “square.”

Approximately how many “square” bales contain the same amount of hay as one “round” bale?

A. 14
B. 17
C. 21
D. 84

C (21): Correct

\[ V \text{ (square bale)} = l \times w \times h = 1.5 \times 1.5 \times 6 = 6.75 \text{ cu. ft.} \]

\[ V \text{ (round bale)} = \pi r^2 h = 3.14 \times 3^2 \times 5 = 3.14 \times 9 \times 5 = 141.3 \text{ cu. ft.} \]

\[ 141.3 / 6.75 = 20.9333 \approx 21 \]
Gene has a cylinder with radius 4 inches and height 2 inches. He cut the cylinder in half along the length of the diameter, as shown in the diagram below.

Radius = 4 then Diameter = 2r = 8
Height = 2

\[ A = l \times w = 8 \times 2 = 16 \text{ in}^2 \]

What is the area of the shaded cross-section?

A. \( 48\pi \) square inches
B. \( 24\pi \) square inches
C. \( 16 \) square inches
D. \( 8 \) square inches

C (16 sq. in.): Correct
Hybrid (composite) Shapes

1. Perimeter
2. Area
Example: Find the perimeter of the composite figure. It is important to make sure that every side has a length before you start adding. Many times some lengths are not given but you are given enough information to find them. For example, to find the perimeter of the following shape you will need to find 2 missing lengths.

The length of the segment on the left, from the top of the figure to the bottom is 10 cm. This means the two vertical segments on the right need to add to 10 cm. To find the length of the red segment, subtract the given vertical segment on the right, 4 cm, from 10 cm. $10 - 4 = 6$. The length of the red segment is 6 cm.

The length of the segment on the bottom is 16 cm. The two horizontal segments that make the top of the figure need to add up to 16 cm. To find the length of the green segment, subtract the given horizontal segment on top, 9 cm, from 16 cm. $16 - 9 = 7$. The length of the green segment is 7 cm.

Fill in the missing lengths on the diagram:

Now you know all of the segment lengths, so add them together to find the perimeter. $P = 6 + 7 + 4 + 16 + 10 + 9 = 52$ cm.
A diagram of the floor plan of a storage room is shown below.

If the ceiling is 12 feet above the floor, what is the capacity of the storage room, in cubic feet?

A. 118 ft³
B. 600 ft³
C. 1,416 ft³
D. 1,800 ft³

C: Correct
The Prom Decoration Committee decided to add the string of lights to its welcome sign, as shown in the drawing.

To the nearest foot, what is the minimum length of the string of lights the committee will need?

A. 15 feet
B. **18 feet**  B (18 ft.): Correct
C. 21 feet
D. 33 feet

Circumference (welcome-1/2 circle) = \( \frac{1}{2} \cdot 3 \cdot \pi = 0.5 \times 3 \times 3.14 = 4.71 \)

Perimeter (entire sign) = 4 + 2.5 + 4.71 + 2.5 + 4 = 17.71 \( \approx 18 \)
Dominic cuts out the largest possible circle from his 2-foot by 2-foot piece of art paper.

Which is a reasonable estimate of the fraction of the art paper that is left over?

A. less than $\frac{1}{4}$  
B. between $\frac{1}{4}$ and $\frac{1}{2}$  
C. between $\frac{1}{2}$ and $\frac{3}{4}$  
D. more than $\frac{3}{4}$

A (less than $\frac{1}{4}$): Correct

Area (square) = $s \times s = 2 \times 2 = 4$ sq ft

Area (circle) = $\pi r^2 = 3.14 \times 1 \times 1 = 3.14$ sq ft

$4.00 - 3.14 = .86$

$.86 / 4 = .215$

and $.215 < .25$
The diagram below shows the dimensions of a wall that needs to be painted. The door represented by the shaded rectangle is not to be painted.

The focus of the item is to determine the area, to the nearest square foot, of the wall to be painted. The response indicates that the area to be painted is approximately 214 ft² with supporting work or explanation.

- \( A = \frac{1}{2} (5 \times 4) = 10 \) ft²
- \( A = 5 \times 2 = 10 \) ft²
- \( A = \frac{1}{2} (11 \times 2) = 11 \) ft²
- \( A = 22 \times 9 = 198 \) ft²
- \( A = 7 \times 3 = 21 \) ft²

\( 6 + 10 + 10 + 11 + 198 - 21 = 214 \) sq. ft.

OR a close approximation based on counting squares.
Marny wants to approximate the amount of wax needed to make a crayon. The dimensions of the crayon are shown below.

\[ V \text{ (cone)} = \frac{1}{3} \pi r^2 h \]
\[ = 0.33 \times 3.14 \times 1^2 \times 3 \]
\[ = 3.1086 \text{ cm}^3 \]
\[ V \text{ (cylinder)} = \pi r^2 h \]
\[ = 3.14 \times 1^2 \times 6 \]
\[ = 18.84 \text{ cm}^3 \]
\[ V \text{ (total)} = V \text{ (cone)} + V \text{ (cylinder)} \]
\[ = 3.11 + 18.84 = 21.95 \text{ cm}^3 \]

About how many cubic centimeters of wax are needed to make this crayon?

A. 18 cm\(^3\)
B. 22 cm\(^3\) **B: Correct**
C. 28 cm\(^3\)
D. 88 cm\(^3\)